Mechanical Vibrations
Homework 1

Please solve problems (1,3,5,7,9,17,24,29,31,34,36) which are selected from chapter two of the text book and submit them as instructed in the course webpage.

2-1 Use the energy method to determine the equations of motion and the natural frequencies of the systems shown in the following figures:

(a) Figure 2-1(b). Assume the mass of the torsional bar k_t is negligible.

(b) Figure 2-1(d). Assume there is no slippage between the cord and the pulley.

(c) Figure 2-1(f). Consider the mass of the uniform rod L.

(d) Figure P2-1(a). Assume there is no slippage between the roller and the surface.

(e) Figure P2-1(b). Assume there is no slippage between the roller and the surface. Neglect the springs k_2 and let the springs k_1 be under initial tension.

(f) Repeat part **e**, including springs k_1 and k_2. Assume all the springs are under initial compression.

(g) Figure P2-1(c). Assume there is no slippage between the pulley and the cord.

2-3 A counter weight in the form of a circular segment as shown in Fig. P2-2(b) is attached to a uniform wheel. The mass of the wheel is 45 kg and that of the segment 4 kg. The wheel-and-segment assembly is swung as a pendulum. If $R_1 = 250$ mm, $R_2 = 230$ mm, and $L = 500$ mm, find the period of the oscillation.

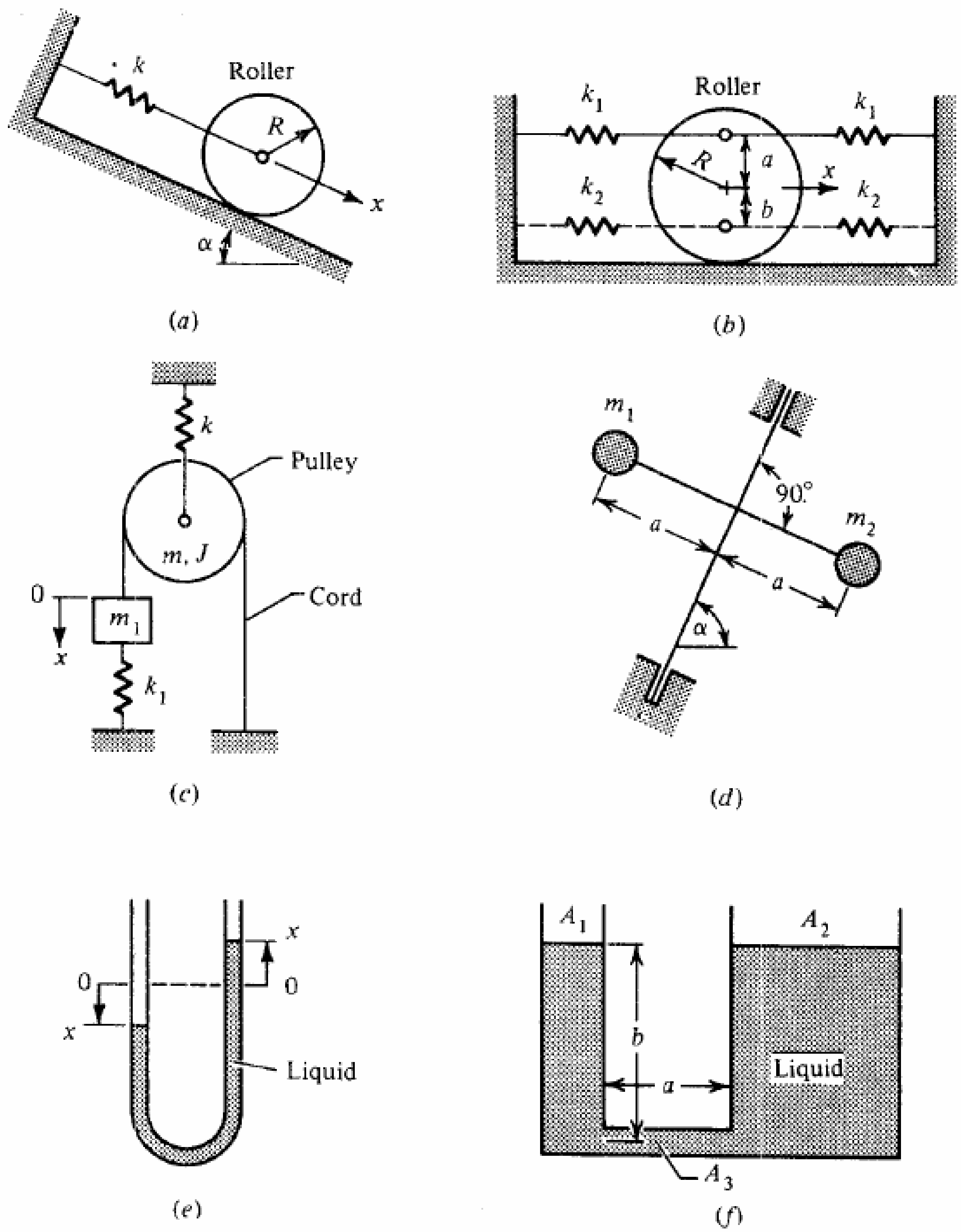
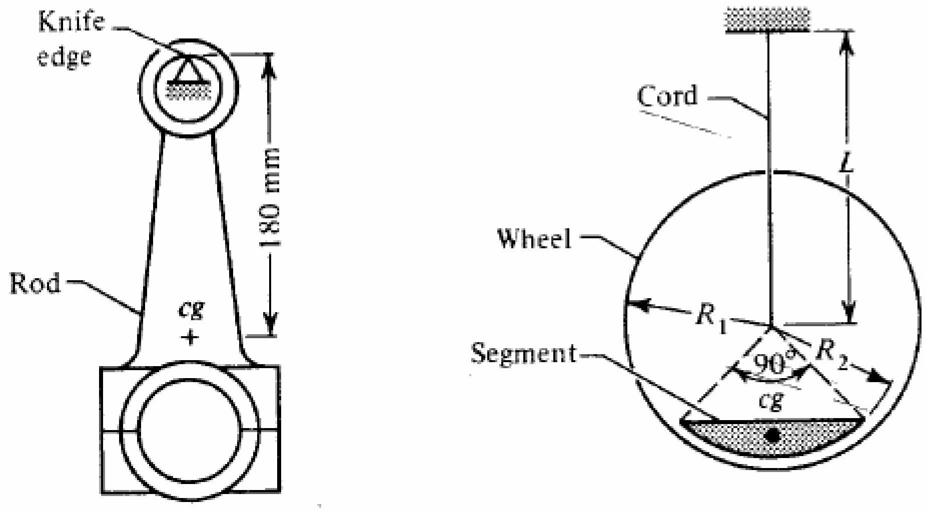
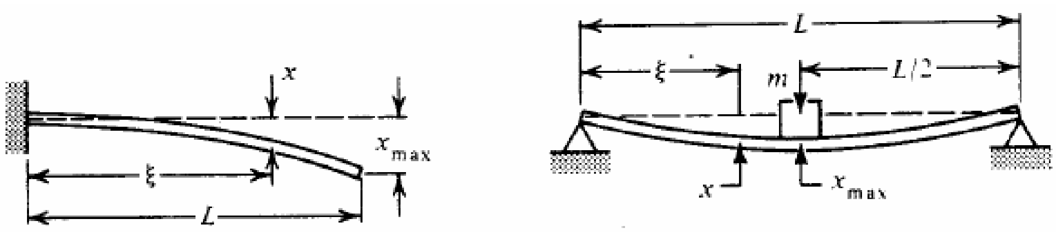


FIG. P2-1. *Vibratory systems.*



(a) Connecting rod (b) Wheel and segment

FIG. P2-2. *Pendulums.*



(a) Cantilever (b) Simply supported beam

FIG. P2-3. *Fundamental frequency of beams.*

2-4 A uniform cantilever beam of ρ mass/length is shown in Fig. P2-3(a). Assume that the beam deflection during vibration is the same as its deflection for a concentrated load at the free end, that is, $x = \frac{1}{2}x_{max}[3(\xi/L)^2 - (\xi/L)^3]$. **(a)** Determine the natural frequency of the beam. **(b)** Define an equivalent mass at the free end of the beam for this mode of vibration.

2-5 Repeat Prob. 2-4 if the deflection curve is assumed as $x/x_{max} = \xi/L$. What is the percentage error in the natural frequency as compared with Prob. 2-4? Note that the assumed deflection curve does not satisfy the boundary condition at the fixed end, since the slope at the fixed end must be zero.

2-6 Repeat Prob. 2-4 if a mass m is attached to the free end of the cantilever.

2-7 A simply supported uniform beam with a mass m attached at midspan is shown in Fig. P2-3(b). The mass of the beam is ρ mass/length. Assume that the deflection during vibration is the same as the static deflection for a concentrated load at midspan. that is, $x = x_{max}[3(\xi/L) - 4(\xi/L)^3]$ for $0 \le \xi \le L/2$. **(a)** Find the fundamental frequency of the system. **(b)** What is the equivalent mass of the beam at $L/2$?

2-9 A uniform bar of ρ mass/length with an attached rigid mass m is shown in Fig. P2-4(a). Assume the elongation of the bar is linear, that is, $x/x_{max} = \xi/L$. Find the frequency for the longitudinal vibration of the bar.

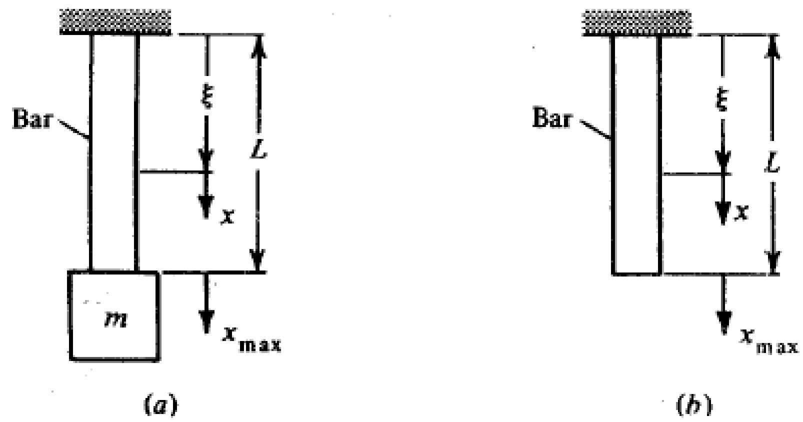


FIG. P2-4. *Fundamental frequency of bars.*

2-17 A machine of 20 kg mass is mounted as shown schematically in Fig. 2-7. If the total stiffness of the springs is 17 kN/m and the total damping is 300 N · s/m, find the motion $x(t)$ for the following initial conditions:

(a) $x(0) = 25$ mm and $\dot{x}(0) = 0$

(b) $x(0) = 25$ mm and $\dot{x}(0) = 300$ mm/s

(c) $x(0) = 0$ and $\dot{x}(0) = 300$ mm/s

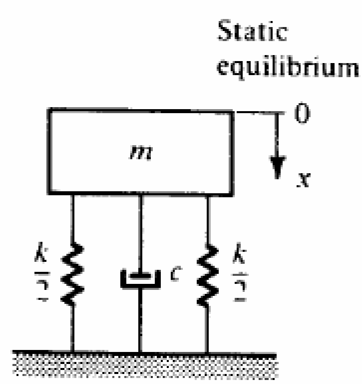


FIG. 2-7. *Damped-free vibration.*

2-24 Derive the equations of motion for each of the systems shown in Fig. P2-5. Derive expressions for the steady-state response of the systems by the mechanical impedance method.

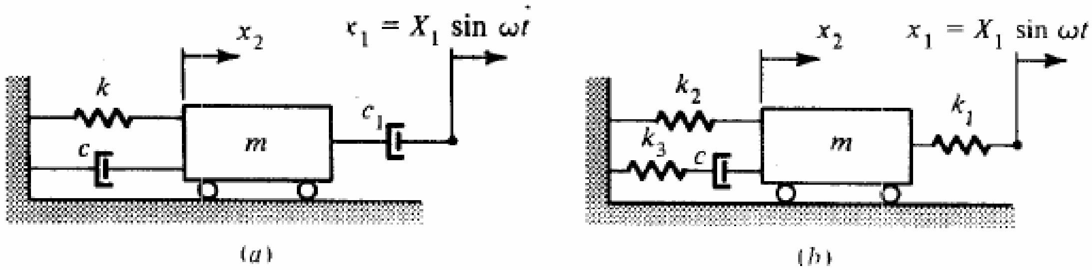


FIG. P2-5. *Vibratory systems.*

2-29 Given the equation of motion of an underdamped system

$$m\ddot{x} + c\dot{x} + kx = F(t) \qquad \text{or} \qquad \ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = F(t)/m$$

derive the equation for the transient response $x(t)$ shown in Eq. (2-74) by (1) multiplying the equation above by $e^{-\zeta\omega_n(t-\tau)} \sin \omega_d(t-\tau)$ and (2) integrating by parts for $0 \le \tau \le t$, that is,

$$x(t) = e^{-\zeta\omega_n t}\left(x_0 \cos \omega_d t + \frac{\dot{x}_0 + \zeta\omega_n x_0}{\omega_d} \sin \omega_d t\right)$$
$$+ \frac{1}{m\omega_d} \int_0^t F(\tau) e^{-\zeta\omega_n(t-\tau)} \sin \omega_d(t-\tau)\, d\tau$$

2-31 Assuming zero initial conditions, find the transient response $x(t)$ of a system described by the equation

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = At$$

by means of Eq. (2-74), where $A = \text{constant}$. Use the classical method to check the answer.

Computer problems:

2-32 Use Matlab program to find the transient response $x(t)$ of the system

$$m\ddot{x} + c\dot{x} + kx = F(t)$$

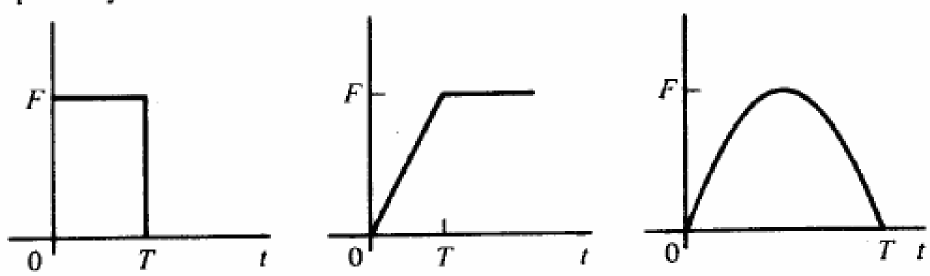
Let $F(t)$ be as shown in Fig. P2-6(a). Choose values for m, c, k, F, and T. Assume appropriate values for the initial conditions x_0 and $\dot{x}_0$. Select about two cycles for the duration of the run and approximately twenty data points per cycle.
Consider the problem in three parts as follows:

(**a**) $F(t) = 0$, $x(0) = x_0$ and $\dot{x}(0) = \dot{x}_0$

(**b**) $F(t) \neq 0$, $x(0) = 0$ and $\dot{x}(0) = 0$

(**c**) $F(t) \neq 0$, $x(0) = x_0$ and $\dot{x}(0) = \dot{x}_0$

Verify from the computer print-out that $x(t)$ in part **c** is the sum of the parts **a** and **b**. In other words, this is to demonstrate Eq. (2-74) in which the response due to the initial conditions and the excitation can be considered separately.



(a) Rectangular pulse (b) Step input with rise time (c) A half sine pulse

FIG. P2-6. *Excitation forces.*

2-33 Repeat Prob. 2-32 for the excitation $F(t)$ shown in Fig. P2-6(b).

2-34 Repeat Prob. 2-32 for the excitation $F(t)$ shown in Fig. P2-6(c).

2-35 Select any transient excitation $F(t)$ and repeat Prob. 2-32.

2-36 It was shown in the pendulum problem in Example 1 that the equation of motion is nonlinear for large amplitudes of vibration. Consider a variation of the pendulum problem in Eq. (2-11).

$$mL^2\ddot{\theta} + c_t\dot{\theta} + mgL \sin\theta = \text{torque}\ (t)$$

or

$$\ddot{\theta} + 2\zeta\omega_n\dot{\theta} + \omega_n^2 \sin\theta = T(t)/mL^2$$

where c_t is a viscous damping factor and $T(t)$ a constant torque applied to the system. Select values for ζ, ω_n, $T(t)$, and the initial conditions $\theta(0)$ and $\dot{\theta}(0)$. Using the fourth-order Runge-Kutta method as illustrated in Fig. 9-1(a), write a program to implement the equation above.